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HEAT AND MASS TRANSFER IN THE COUETTE FLOW OF A PARTIALLY IONIZED DIATOMIC GAS

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Abstract—The Couette flow of a partially ionized symmetric diatomic gas is studied for both chemical equilibrium and chemically frozen flow; the results represent a direct generalization of Couette flow with dissociation only. Emphasis is focused on illuminating the role of ionization through the use of this relatively simple geometry, and the introduction of a new ternary model for diffusion. The gas mixture is considered to consist of symmetric diatomic molecules, atoms, ions (atomic) and electrons, except for diffusion effects where the ions and electrons are assumed to diffuse together as one unit (ambipolar diffusion). The theory is limited to flows where the ion or electron number density, though significant, is small relative to the neutral particles and the analysis is carried out in a linearized fashion with respect to the ion-electron mass fractions. The linearized equations are solved analytically and numerical results are presented for nitrogen. Ionization is found to produce an ion-electron mass flux flowing from the upper "hot" wall to the lower "cool" wall; when the gas phase is in chemical equilibrium ionization causes the atomic mass flux to split into two streams, one diffusing toward the upper "hot" wall and the other towards the lower "cool" wall. Ionization has a marked effect on temperature distribution and the recovery enthalpy. The heat transfer rate increases with the degree of ionization.

NOMENCLATURE

C_f,	skin friction coefficient;
c_i,	mass fraction of species "*i*";
c_{pf},	"frozen" specific heat of gas mixture;
$\mathcal{D}_{ij}$,	binary diffusion coefficient;
h,	enthalpy per unit mass of mixture;
h_i,	enthalpy per unit mass of species "*i*";
h_0^D,	dissociation energy per unit mass of atoms and ions;
h_0^I,	ionization energy per unit mass of ions;
k,	Boltzmann's constant;
K_D,	dissociation equilibrium coefficient;
K_I,	ionization equilibrium coefficient;
L,	Lewis number, defined in what follows equation (3.11);
m_i,	mass of particle "*i*";
N_i,	mass flux of species "i" into the lower wall;
Nu,	Nusselt number, $= St\, Re_\delta Pr_f$;
n,	total number density of the mixture;
n_i,	number density of species "*i*";
n_H,	number density of heavy particles, $n_M + n_A + n_I$;
Pr_f,	"frozen" Prandtl number;
$(-q_w)$,	heat flux into the lower wall;
Re_δ,	Reynolds number, $= \rho_\delta u_\delta \delta/\mu_\delta$;
St,	Stanton number, $= - q_w/[\rho_\delta u_\delta(h_r - h_w)]$;
S_{ij},	diffusion cross-section between particles of species "*i*" and species "*j*";
V_i,	diffusion velocity of species "*i*";
u,	mass average velocity;
x, y,	distances along and normal to the flow;

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Greek symbols

Φ,	vibrational contribution to molecule specific heat;
ψ,	vibrational contribution to molecule enthalpy;
Γ_{Aw},	ratio of the number of atoms converted at the wall into molecules to the total number of atoms striking the wall per unit time;
Γ_{Rw},	ratio of the number of ion-electron particles converted at the wall to neutral particles to the total number of ion-electron particles striking the wall per unit area per unit time;
δ,	distance between plates;
κ,	"frozen" thermal conductivity;
κ_r,	"reaction" thermal conductivity;
κ_{eq},	"equilibrium" thermal conductivity;
$\dot{\omega}_i$,	mass rate of formation of species "i";
ρ,	mass density of mixture;
θ,	recovery enthalpy factor,
μ,	viscosity of the mixture.

Subscripts

A,	atoms;
E,	electron;
I,	ion;
M,	molecule;
R,	ion-electron;
e,	chemical equilibrium;
r,	recovery (insulated lower wall);
w,	lower wall;
δ,	upper wall;
v,	vibration.

1. INTRODUCTION

IN THIS paper we study the effects of ionization, using a ternary model of diffusion (molecules, atoms, and an ion-electron specie determine the effects of diffusion) on a dissociating and ionizing symmetric diatomic gas in a Couette flow. This ternary model of diffusion has not, to the authors' knowledge, been used before in Couette or boundary layer types of flow. Its use represents a generalization of Clarke's [1] work (dissociation; binary model) to include partial ionization (dissociation and ionization; ternary model). The use of this model in obtaining an analytic solution shall be presented.

The simplifications resulting from the geometry of this type of flow were exploited by Illingworth [2] to solve the compressible flow problem exactly, and by Clarke [1] and Enkenhus [3] to study the effects of dissociation (using a binary model of diffusion) on the flow of a symmetric diatomic gas. Leipmann and Bleviss [4] considered the Couette flow of a dissociated gas, and indicated how the effect of ionization, on a gas in chemical equilibrium, can be taken into account. Couette flow was also used by Bleviss [5, 6] to study the effects of magnetic and electric fields on the flow of an electrically conducting gas in chemical equilibrium. Chung [7] studied the electrical characteristics of a slightly ionized monoatomic gas in a Couette flow.

The motivation for treating this type of flow again is the ability to obtain analytical solutions in a form which explicitly reveals the effects of ionization. We intend to exploit these results to disclose in some detail the influence of ionization (and the artificial suppression of ionization) on the flow of a dissociating and ionizing symmetric diatomic gas.

The effect of ionization on heat transfer rates in stagnation point flow has recently been studied by several authors. Adams [8] used the dissociated gas theory of Fay and Riddel [9] with a correlation factor to account for ionization. Hoshizaki [10], Pallone and Von-Tassel [11] and Cohen [12] considered only the chemical equilibrium case of a partially ionized diatomic gas. Solutions were obtained by numerically integrating the governing equations using the total thermodynamic and transport properties of the gas. Scala and Warren [13] also treated the chemical equilibrium flow of ionized nitrogen in the stagnation regime. The gas was considered to be a four component mixture: molecules, atoms, ions and electrons. There is an indication that they may have assumed too small a diffusion cross-section between ions and electrons. Fay and Kemp [14] and Fenster [15] used a binary model of diffusion for the purpose of evaluating the diffusive mass fluxes. Fay and Kemp considered atoms, ions and electrons to diffuse together as one particle; molecules, however, were assumed to have a different diffusion velocity. Fenster used this same model when the flow is chemically frozen; when the flow is in chemical equilibrium he assumed that air, before it gets ionized, consists of atoms and molecules; when it is ionized there are atoms and ion-electron particles only.

Scala and Warren's calculations show that the heat transfer rate increases by more than a factor of two as compared with that calculated by extrapolating the dissociated gas theory. The other authors' results predict that ionization increases the heat transfer rate, over that calculated by the extrapolated dissociated gas theory, by only 30 per cent or less.

These divergent results suggest that further study is desirable; furthermore, the methods of treatment mentioned above do not disclose explicitly the role played by the diffusion of the ionized particles in the flow, except for the work of Fenster [15] for a binary model. By using the total thermodynamic and transport properties, the individual species mass fluxes, through which the diffusion role of each type of particle in the gas mixture may be examined, cannot be obtained. The binary model of diffusion used by Fay and Kemp suppresses the diffusion of the ionized particles (ions and electrons) relative to the atoms; in particular, when the gas is at sufficiently high temperatures where there are only a few molecules, it effectively eliminates diffusion completely.

The partially ionized symmetric diatomic gas we are considering is a mixture of four different kinds of particles: molecules, atoms, ions (atomic) and electrons. Because of their small mass, electrons should have the largest diffusion velocity; however, due to Coulomb forces between the charged particles, electrons are decelerated by the heavier, slower diffusing ions. In ionized gas mixtures, where there are no self generated nor imposed electric fields, ions and electrons have been considered to diffuse together as if they were one particle, which is known as "ambipolar" diffusion. This consideration is valid for phenomena taking place at a characteristic distance greater than the Debye shielding distance;† within the Debye distance electrons diffuse with a different velocity than ions. In this paper, the Debye shielding distance is assumed much smaller than the characteristic length of the flow. Thus, when considering the effects of diffusion on such flows, the ionized symmetric diatomic gas will be considered to consist of molecules, atoms and "ion-electron" particles; the "ion-electron" particles diffuse as one unit. For effects other than diffusion, all four particles are considered. The ion-electron particles are designated by the subscript "R".

2. FLOW EQUATIONS, MULTICOMPONENT DIFFUSION COEFFICIENTS AND DIFFUSIVE MASS FLUXES

The governing equations for a multicomponent gas mixture in a Couette flow are:

$$\mathrm{d}(\rho c_i V_i)/\mathrm{d}y = \dot{\omega}_i, \tag{2.1}$$

$$\mathrm{d}[\mu(\mathrm{d}u/\mathrm{d}y)]/\mathrm{d}y = 0, \tag{2.2}$$

$$\mathrm{d}[\kappa(\mathrm{d}T/\mathrm{d}y) - \rho \sum_i h_i c_i V_i]/\mathrm{d}y + \mu(\mathrm{d}u/\mathrm{d}y)^2 = 0, \tag{2.3}$$

where in the energy equation (2.3) radiation has been neglected.

The rate of species mass formation $\dot{\omega}_i$ is zero in gas phase when the flow is chemically frozen; for equilibrium flow its value need not be evaluated explicitly as will be seen in section 3.1.

The diffusion velocity V_i must be expressed in terms of variables such as ρ, c_i, etc., and their gradients. For a multicomponent gas mixture the diffusion velocity for each of the species in

† The Debye [16] shielding distance = $6{\cdot}9(T_E/n_E)^{\frac{1}{2}}$ cm; for $p = 1{\cdot}0$ atm and $T_E = T = 12\,000$°K it is equal to $2{\cdot}91 \times 10^{-6}$ cm.

the mixture can be obtained from the species mass flux equation which, when neglecting thermal diffusion, is [17]

$$\rho_i V_i = (n^2/\rho) \sum_{j \neq i} m_i m_j D_{ij} [\mathrm{d}(n_j/n)/\mathrm{d}y]. \tag{2.4}$$

Neglect of thermal diffusion is reasonable when temperature gradients are not too large, and will enable us to compare our results with other work [14].

Since $\sum_i \rho_i V_i = 0$, only the mass fluxes of atoms and "ion-electron" particles need be obtained for the ternary model considered here. In order to do this, we note that the mass of the combined "ion-electron" particle is $m_R = m_I + m_E = m_A$; since the effects of diffusion are to be accounted for by a 3-component model we write

$$\begin{aligned} \rho &= n\bar{m} = n_M m_M + n_A m_A + n_R m_A, \\ \sum_{i=1}^{3} c_i &= \sum_{i=1}^{3} \rho_i/\rho = c_M + c_A + c_R = 1, \qquad (2.5) \\ 1/\bar{m} &= n/\rho = \sum_{i=1}^{3} n_i/\rho = \sum_{i=1}^{3} c_i/m_i. \end{aligned}$$

Using equation (2.5) and noting that $m_M = 2m_A$, one obtains

$$\begin{aligned} \mathrm{d}(n_M/n)/\mathrm{d}y &= -(\bar{m}^2/m_A m_M)(\mathrm{d}c_A/\mathrm{d}y + \mathrm{d}c_R/\mathrm{d}y), \\ \mathrm{d}(n_A/n)/\mathrm{d}y &= (\bar{m}^2/m_A m_M)[(1 + c_R)(\mathrm{d}c_A/\mathrm{d}y) \\ &\qquad - c_A(\mathrm{d}c_R/\mathrm{d}y)], \\ \mathrm{d}(n_R/n)\,\mathrm{d}y &= (\bar{m}^2/m_A m_M)[-c_R(\mathrm{d}c_A/\mathrm{d}y) \\ &\qquad + (1 + c_A)(\mathrm{d}c_R/\mathrm{d}y)]; \end{aligned} \tag{2.6}$$

of course, $\bar{m}^2$ is not constant.

Substituting equation (2.6) into equation (2.4) and using equation (2.5), the atomic and the "ion-electron" mass fluxes become

$$\begin{aligned} \rho c_A V_A &= -\rho[D_{AM} + (1/2)\, c_R D_{AR}](\mathrm{d}c_A/\mathrm{d}y) \\ &\quad + \rho[(1/2)\, D_{AR}(1 + c_A) - D_{AM}](\mathrm{d}c_R/\mathrm{d}y), \\ \rho c_R V_R &= \rho[(1/2)(1 + c_R) D_{RA} - D_{RM}](\mathrm{d}c_A/\mathrm{d}y) \\ &\quad - \rho[(1/2)\, c_A D_{RA} + D_{RM}](\mathrm{d}c_R/\mathrm{d}y). \end{aligned} \tag{2.7}$$

Theories dealing with gas transport properties consider the binary diffusion coefficient $\mathscr{D}_{ij}$ rather than the multicomponent diffusion coefficient D_{ij}. We, therefore, express equation (2.7) in terms of $\mathscr{D}_{ij}$; this can be done for a ternary gas by using (see Hirschfelder *et al.* [18])

$$\begin{aligned} D_{12} = \mathscr{D}_{12}\{1 + n_3[(m_3/m_2)\,\mathscr{D}_{13} \\ - \mathscr{D}_{12}]/(n_1\mathscr{D}_{23} + n_2\mathscr{D}_{13} + n_3\mathscr{D}_{12})\} \end{aligned} \tag{2.8}$$

notice that $D_{12} \neq D_{21}$ but $\mathscr{D}_{12} = \mathscr{D}_{21}$. Now substitute equation (2.8) into equation (2.7); the resulting expressions for the mass fluxes are in terms of $\mathscr{D}_{AR}$, $\mathscr{D}_{AM}$, $\mathscr{D}_{RM}$, c_A and c_R. In addition to being complex, interpretation of these expressions is difficult. They can, however, be further simplified.

The binary diffusion coefficient is given in terms of the diffusion cross-section S_{ij} by [19]

$$\mathscr{D}_{ij} = (3/8)\,[\pi kT/2)(m_i + m_j)/m_i m_j]^{\frac{1}{2}}/nS_{ij}. \tag{2.9}$$

The diffusion cross-sections of ion-atom (S_{IA}) and ion-molecule (S_{IM}) have been calculated by Hansen [20]; he found that $S_{AI} = S_{AA}$. Hence, we shall consider $S_{AR} = S_{AA}$, $S_{RM} = S_{AM}$. Furthermore, the ratio S_{AR}/S_{AM} was calculated from Hansen's data and found to vary little in the range where ionization is present ($S_{AR}/S_{AM} \cong 0{\cdot}577$); we shall take this ratio to be $1/\sqrt{3}$. Then equation (2.9) yields

$$\begin{aligned} \mathscr{D}_{AR}/\mathscr{D}_{AM} &= (1 + c_A - c_R)/(c_A + c_R), \\ \mathscr{D}_{RM}/\mathscr{D}_{AM} &= (1 + c_A - c_R)/(1 + c_R - c_A); \end{aligned} \tag{2.10}$$

$\mathscr{D}_{AR}$ and $\mathscr{D}_{RM}$ are now related to $\mathscr{D}_{AM}$. Using equation (2.10) in equation (2.8) the multicomponent diffusion coefficients can be expressed in terms of $\mathscr{D}_{AM}$, c_A, and c_R. When these coefficients are substituted into equation (2.4), the atomic and "ion-electron" mass fluxes are obtained.

The resulting expressions are, however, still quite complex. Further simplification is achieved by noting that in the range of temperature and pressure we will consider ($T < 12\,000$°K, $p > 0{\cdot}1$ atm) ionization is dominated by dissociation. Therefore, c_R^2 and higher powers of c_R are much smaller than c_A and c_R; the resulting linearized (with respect to c_R and its derivatives) expressions for the atomic and the "ion-electron" mass fluxes are

$$\begin{aligned}\rho c_A V_A = \rho \mathscr{D}_{AM}\{ &-[1 + 2c_R(1 - c_A + c_A^2 \\ &+ c_A^3)/(1 - c_A + c_A^3)]\,(\mathrm{d}c_A/\mathrm{d}y) \\ &+ [2(c_A + c_A^4)/(1 - c_A + c_A^2 \\ &\qquad + 3c_A^3)]\,(\mathrm{d}c_R/\mathrm{d}y)\},\end{aligned} \qquad (2.11a)$$

and

$$\begin{aligned}\rho c_R V_R = \rho \mathscr{D}_{AM}\{&[2c_R(1 - c_A^2)/(1 - c_A \\ &+ c_A^2 + 3c_A^3)]\,(\mathrm{d}c_A/\mathrm{d}y) - [(1 + c_A)^3/(1 - c_A \\ &\qquad + c_A^2 + 3c_A^3)]\,(\mathrm{d}c_R/\mathrm{d}y)\};\end{aligned} \qquad (2.11b)$$

the details may be found in [21].

At low temperatures, where the gas is dissociated only, $c_R \cong 0$ and equation (2.11b) shows that $\rho c_R V_R \cong 0$; the atomic mass flux [equation (2.11a)] reduces exactly to that of a dissociated gas (see, for example, Clarke [1]). Thus, the diffusive mass flux obtained by using Ficks law (binary gas) is a special case of equation (2.11).

The qualitative effect of ionization on the species mass fluxes may now be deduced. It is readily seen that the polynomials containing c_A in equation (2.11) are positive and finite for $0 \leqslant c_A \leqslant 1$. First, consider the case when the gas is in chemical equilibrium. Start from the upper wall (corresponding to the outer region of a boundary layer) and move towards the lower wall (decreasing y); at the "hot" upper wall there exist atoms, ions, electrons and a negligible number of molecules. Moving away from the "hot" upper wall towards the "cool" lower wall, electrons and ions recombine forming atoms; molecules, however, are not formed as fast as atoms because of the high temperature that exists in this region. Hence, $\mathrm{d}c_R/\mathrm{d}y$ is positive and $\mathrm{d}c_A/\mathrm{d}y$ is negative; equation (2.11a) indicates that the atomic mass flux is towards the upper wall; the ion-electron mass flux is towards the lower wall, as can be seen from equation (2.11b). The directions of these fluxes persist as we move away from the upper "hot" wall until the atom mass fraction reaches a maximum; at this position $\mathrm{d}c_A/\mathrm{d}y = 0$, and if $\mathrm{d}c_R/\mathrm{d}y \neq 0$ the atom mass flux is still towards the upper wall and the "ion-electron" mass flux is towards the lower wall. Moving, further, towards the "cool lower wall, this position of maximum c_A is followed by a region where the small number of ion-electron particles that may be left in the flow are neutralized; the atoms, by then, have started to recombine forming molecules and, hence, $\mathrm{d}c_A/\mathrm{d}y$ becomes positive and the atomic mass flux is now towards the lower wall (see equation 2.11a). At very low temperatures, close to the lower wall, all atoms have recombined leaving only molecules which are diffusing towards the upper wall.

We now compare the above results with the case where ionization is suppressed; as in the previous paragraph the flow is assumed to be in chemical equilibrium. Near the upper wall the temperature is high enough so that the gas is completely dissociated. The gas may stay in this state in the layer contiguous with the upper wall before atoms start to recombine forming molecules. In this region $\mathrm{d}c_A/\mathrm{d}y \cong 0$ and there is no atomic mass flux. Moving towards the lower wall the temperature decreases until it is low enough for atoms to recombine forming molecules, and $\mathrm{d}c_A/\mathrm{d}y > 0$; the atomic mass flux is now towards the lower wall. This trend continues until all atoms recombine and the gas close to the lower wall consists only of molecules diffusing back towards the upper wall, there is no local maximum in the concentration of atomic species, as in the case when ionization is included. Thus, ionization introduces an ion-electron mass flux diffusing towards the lower "cool" wall and causes the atomic mass flux to split into two streams, one flowing towards the upper wall and the other flowing towards the lower wall.

When no chemical reaction takes place in the layer, i.e., the flow is chemically frozen, the species conservation equation shows that the mass flux for every kind of particle in the flow is constant. If the lower wall is catalytic, mass fraction gradients exist in the flow and there are atomic and "ion-electron" mass fluxes towards the lower wall. When ionization is suppressed, the upper wall is generating atoms only, and the flow is devoid of any ionized particles.

3. PARTICLE DISTRIBUTIONS

3.1 *Particle concentration in the gas phase*

Because $\sum_{i=1}^{3} c_i = 1$, the mass fraction distribution of only two species need be obtained; those of atoms and ion-electron particles are considered here. Furthermore, only the two limiting cases of chemical equilibrium and frozen flow will be discussed. Chemical equilibrium flow and chemically frozen flow serve as limits for the complex chemically non-equilibrium flow. These two cases have been used in earlier studies [1, 14] and are once more utilized here.

First consider the case when the gas phase is in chemical equilibrium. It shall be assumed that only the following reactions take place;

$$\begin{aligned} &\text{particles} + A_2 \rightleftarrows 2A + \text{particles}, \\ &\text{particles} + A \rightleftarrows A^+ + E + \text{particles}. \end{aligned} \tag{3.1}$$

The laws of mass action for the reactions of equation (3.1) are well known (see Drellishak [22], for example) and for constant pressure Couette flow may be written as

$$n_A^2/n_M = K_D \tag{3.2}$$

$$n_I n_E/n_A = K_I,$$

where $n_{A_2} = n_M, n_A^+ = n_I$. Since $n_R = n_I = n_E$ and $p = nkT$, equation (3.2) yields,

$$\alpha^2 c_A^2 = 1 + 2c_R - 4c_A c_R - 3c_R^2, \tag{3.3}$$

$$c_R^2 = \zeta[c_A + c_A^2 + 3c_A c_R], \tag{3.4}$$

where

$$\alpha = (1 + 4p/kTK_D)^{\frac{1}{2}}, \zeta = kTK_I/2p.$$

Equations (3.3) and (3.4) are to be solved for c_A and c_R in terms of α and ζ. Exact analytical solutions cannot be found. However, solutions for c_A and c_R can be obtained in terms of α and ζ by taking advantage of the small order of magnitude of c_R. It can be seen from equation (3.4) that small c_R implies small ζ.

Since c_A and c_R vary between zero and unity the only physical roots of equations (3.3) and (3.4), respectively, are

$$c_A = (1/2\alpha^2)\,[-4c_R + 2\alpha(1 + 2c_R - 3c_R^2 + 4c_R^2/\alpha^2)^{\frac{1}{2}}],$$

$$c_R = (1/2)\,\{3\zeta c_A + [(3\zeta c_A)^2 + 4\zeta(c_A + c_A^2)]^{\frac{1}{2}}\};$$

expanding in terms of c_R and ζ respectively one gets

$$c_A = 1/\alpha + (1/\alpha - 2/\alpha^2)\,c_R + (2/\alpha^3 - 7/4\alpha)\,c_R^2 + \dots, \tag{3.5}$$

$$\begin{aligned} c_R = {} & (c_A + c_A^2)^{\frac{1}{2}}\,\zeta^{\frac{1}{2}} + (3/2)\,c_A\zeta \\ & + (9/8)\,c_A^2(c_A + c_A^2)^{-\frac{1}{2}}\,\zeta^{\frac{3}{2}} \\ & - (1/8)\,[(9/4)\,c_A^2(c_A + c_A^2)^{-2}]^2\,(c_A + c_A^2)^{\frac{1}{2}}\,\zeta^{\frac{5}{2}} + \dots. \end{aligned} \tag{3.6}$$

Equations (3.5) and (3.6) express c_A and c_R as

$$c_A(\alpha, c_R) = \sum_i a_i(\alpha)\,c_R^i,$$

and

$$c_R(c_a, \zeta^{\frac{1}{2}}) = \sum_i b_i(c_A)\,(\zeta^{\frac{1}{2}})^i.$$

We wish to obtain c_A and c_R as functions of α and ζ. This is done by constructing a Taylor's series expansion for $c_A(\alpha, \zeta^{\frac{1}{2}})$ and $c_R(\alpha, \zeta^{\frac{1}{2}})$; the required coefficients may be computed from equations (3.5) and (3.6); one obtains

$$\begin{aligned} c_A = {} & 1/\alpha + (1/\alpha - 2/\alpha^2)(1/\alpha + 1/\alpha^2)^{\frac{1}{2}}\zeta^{\frac{1}{2}} \\ & + [(1/\alpha - 2/\alpha^2).(1 + 2/\alpha) + 3/\alpha \\ & + (2/\alpha^3 - 7/4\alpha)(1/\alpha + 1/\alpha^2)]\,\zeta + \dots, \end{aligned} \tag{3.7}$$

$$c_R = (1/\alpha + 1/\alpha^2)^{\frac{1}{2}}\zeta^{\frac{1}{2}} + [(1/\alpha - 2/\alpha^2)(1 + 2/\alpha) + 3/\alpha]\,\zeta + \dots, \tag{3.8}$$

where equations (3.7) and (3.8) are valid for ζ sufficiently small.

It may be noted that when the gas is dissociated only, $\zeta = 0$; thus, $c_R = 0$ and equation (3.7) gives $c_A = 1/\alpha$; which is the case for a dissociating diatomic gas [23].

Having obtained the mass fractions in terms of α and ζ, which are functions of p and T, the mass fraction distribution in a Couette (constant pressure) flow can be evaluated once the temperature distribution is obtained. This can be achieved, as shall be described later, by solving equations (2.2) and (2.3).

When the characteristic time for chemical reaction is much smaller than a characteristic time for diffusion the chemical reactions in the gas phase may be neglected; the flow is then called chemically frozen. The species conservation equation becomes $\mathrm{d}(\rho c_i V_i)/\mathrm{d}y = 0$ from which

$$\mathrm{d}(\rho c_A V_A)/\mathrm{d}y = \mathrm{d}(\rho c_R V_R)/\mathrm{d}y = 0. \qquad (3.9)$$

For convenience in integrating equations (2.11) and (3.9) we change the independent variable from y to u (noting that $\mu \mathrm{d}u/\mathrm{d}y = \tau_w =$ constant); integrating once, we obtain

$$\begin{aligned}(\rho \mathscr{D}_{AM}/\mu)\{[1 + 2c_R(1 - c_R + c_A^2 + c_A^3)/(1 - c_A \\ + c_A^2 + 3c_A)]\,(\mathrm{d}c_A/\mathrm{d}u) - [2(c_A + c_A^4)/(1 - c_A \\ + c_A^2 + 3c_A^3)]\,(\mathrm{d}c_R/\mathrm{d}u)\} = K_1', \qquad (3.10)\end{aligned}$$

$$\begin{aligned}(\rho \mathscr{D}_{AM}/\mu)\{[(1 + c_A)^3/(1 - c_A + c_A^2 \\ + 3c_A^3)]\,(\mathrm{d}c_R/\mathrm{d}u) - [2(1 - c_A^2)/(1 - c_A + c_A^2 \\ + c_A^3)]\,(\mathrm{d}c_A/\mathrm{d}u)\} = K_2'. \qquad (3.11)\end{aligned}$$

Together with Fay and Kemp [14] define a Lewis number $L = n_H m_A \mathscr{D}_{AM} c_{PA}/\kappa_A$, which is also considered constant ($L = 0{\cdot}6$) in this analysis. The viscosity of the ionized gas [see Appendix, equation (A.7)] is taken to be

$$\mu = \mu_A[0{\cdot}82 + 1{\cdot}18(c_A + c_R)]/(1 + c_A + 3c_R).$$

When this is substituted into equations (3.10) and (3.11), and after linearizing with respect to c_R, they become

$$\begin{aligned}\{(1 + c_A) + c_R[3 + 2(1 - c_A + c_A^2 + c_A^3)/(1 \\ - 2c_A + 3c_A^2)]\}\,(\mathrm{d}c_A/\mathrm{d}u) - 2[(c_A + c_A^4)/(1 \\ - 2_A + 3c_A^2)]\,(\mathrm{d}c_R/\mathrm{d}u) = K_1[0{\cdot}82 \\ + 1{\cdot}18c_A)(1 + c_A + c_R) + 1{\cdot}18c_R(1 + c_A)],\end{aligned} \qquad (3.12)$$

and

$$\begin{aligned}[(1 + c_A)^3/(1 - 2c_A + 3c_A^2)]\,(\mathrm{d}c_R/\mathrm{d}u) \\ - 2[c_R(1 - c_A^2)/(1 - 2c_A + 3c_A^2)](\mathrm{d}c_A/\mathrm{d}u) \\ = K_2[0{\cdot}82 + 1{\cdot}18\,c_A)(1 + c_A + c_R) \\ + 1{\cdot}18\,c_R(1 + c_A)] \qquad (3.13)\end{aligned}$$

$$K_1 \equiv K_1'/3L \text{ and } K_2 \equiv K_2'/3L.$$

To obtain the atom and ion-electron particle distributions one must solve, simultaneously, equations (3.12) and (3.13). Analytical solutions will now be developed in such a manner as to take advantage of the assumption of small c_R. The form of solution is taken to be

$$c_A = c_{A0} + \varepsilon c_{A1} + \varepsilon^2 c_{A2} + \ldots \text{ and } c_R = \varepsilon c_{R1} + \varepsilon^2 c_{R2} + \ldots,$$

where ε is a symbol denoting the order of magnitude of the term it is associated with; ε will be assumed small enough so that

$$\begin{aligned}c_A &= c_{A0} + \varepsilon c_{A1}, \\ c_R &= \varepsilon c_{R1}.\end{aligned} \qquad (3.14)$$

Inspection of equations (3.12) and (3.13) suggests that to order ε, $K_1 = K_{10} + \varepsilon K_{11}$, and $K_2 = \varepsilon K_{21}$. Substituting these constants as well as equations (3.14) into equations (3.12) and (3.13), the governing equations for c_{A0}, εc_{R1} and εc_{A1} become the consecutive set of linear differential equations

$$\mathrm{d}c_{A0}/\mathrm{d}u = K_{10}(0{\cdot}82 + 1{\cdot}18\, c_{A0}), \tag{3.15a}$$

$$\mathrm{d}(\varepsilon c_{R1})/\mathrm{d}c_{A0} - (1 - c_{A0})\,\varepsilon c_{R1}/(1 + c_{A0})^2 = \varepsilon K_{21} K_{10}^{-1}(1 - 2c_{A0} + c_{A0}^2)/(1 + c_{A0})^2, \tag{3.15b}$$

and

$$\begin{aligned}\mathrm{d}(\varepsilon c_{A1})/dc_{A0} - 1{\cdot}18\,\varepsilon c_{A1}/(0{\cdot}82 + 1{\cdot}18\, c_{A0}) &= \varepsilon K_{11}/K_{10} + 2\varepsilon K_{21}K_{10}^{-1}(c_{A0} + c_{A0}^4)/(1 + c_{A0})^3 \\ &\quad + \varepsilon c_{R1}[1{\cdot}18/(0{\cdot}82 + 1{\cdot}18\, c_{A0}) - 2/(1 + c_{A0}) - 2(1 + c_{A0}^2)/(1 + c_{A0})^2].\end{aligned} \tag{3.15c}$$

Equation (3.15a) has been used to obtain equation (3.15b) and both equations (3.15a) and (3.15b) were used to arrive at equation (3.15c). The boundary conditions for c_{A0}, εc_{A1} and εc_{R1} are expressed as follows:

$$\text{at } y = 0, u = 0, c_{A0} = c_{A0w}, \varepsilon c_{A1} = \varepsilon c_{A1w}, \varepsilon c_{R1} = \varepsilon c_{R1w};$$

$$\text{at } y = \delta, u = u_\delta, c_{A0} = c_{A0\delta}, \varepsilon c_{A1} = \varepsilon c_{A1\delta}, \varepsilon c_{R1} = \varepsilon c_{R1\delta}.$$

The solutions for equation (3.15) are

$$c_{A0} = (0{\cdot}695 + c_{A0w})\,[(0{\cdot}82 + 1{\cdot}18\, c_{A0\delta})/(0{\cdot}82 + 1{\cdot}18\, c_{A0w})]\,u/u^{\delta} - 0{\cdot}695, \tag{3.16a}$$

$$\varepsilon c_{R1} = \varepsilon K_{21} K_{10}^{-1} F + \varepsilon K_4\{\exp.\,[-4/(1 + c_{A0})]\}/(1 + c_{A0})^2, \tag{3.16b}$$

$$\varepsilon c_{A1} = (0{\cdot}82 + 1{\cdot}18\, c_{A0})\,(\varepsilon K_5 + \varepsilon K_{11} I + \varepsilon K_4 \Phi_1 - \varepsilon K_{21} K_{10}^{-1} \Phi_2) - \varepsilon c_{R1}, \tag{3.16c}$$

where

$$K_{10} = \frac{1}{1{\cdot}18 u_\delta} \ln\,[(0{\cdot}82 + 1{\cdot}18\, c_{A0\delta})/(0{\cdot}82 + 1{\cdot}18\, c_{Aw})\},$$

$$\begin{aligned}F = (1 + c_{A0}) - 2 - 2/(1 + c_{A0}) - 8(1 + c_{A0})^{-1} \exp.\,[-4/(1 + c_{A0})\} \\ \times\ [\ln(1 + c_{A0}) - \sum_{n=1}^{\infty} 4^n/(1 + c_{A0})^n\, nn!],\end{aligned}$$

$$I = (1{\cdot}18\, K_{10})^{-1} \ln(0{\cdot}82 + 1{\cdot}18\, c_{A0}),$$

$$\Phi_1 = (1{\cdot}18)^{-1} \sum_{n=0}^{\infty} (-1)^n\,(4/n!) \sum_{s=0}^{\infty} [2(0{\cdot}36/1{\cdot}18)^s\,(n + s + 3)^{-1}\,(1 + c_{A0})^{-n-s-3}],$$

$$\begin{aligned}\Phi_2 &= (1{\cdot}18)^{-1} \ln(0{\cdot}82 + 1{\cdot}18\, c_{A0}) - (8/0{\cdot}36) \ln\,[(0{\cdot}82 + 1{\cdot}18\, c_{A0})/(1 + c_{A0})] \\ &\quad + (16/1{\cdot}18) \sum_{n=0}^{\infty} (-1)^n\,(4^n/n!) \sum_{s=0}^{\infty} (0{\cdot}36/1{\cdot}18)^s \sum_{m=1}^{\infty} (4^m/mm!) \\ &\quad \times \{[\ln(1 + c_{A0}) + (n + s + 2)^{-1}]/(n + s + 2)\,(1 + c_{A0})^{n+s+2} \\ &\qquad - (n + s + m + 2)^{-1}\,(1 + c_{A0})^{-n+s+m+2}\},\end{aligned}$$

εK_{21}, εK_4, εK_5, and εK_{11} are complicated, but known, functions of the upper plate velocity and concentrations at the boundaries. They are given in [21]. The evaluation of $c_{A0\delta}$, c_{A0w}, $\varepsilon c_{A1\delta}$, εc_{A1w}, c_{A1w}, $c_{R\delta}$ and c_{Rw} shall be discussed.

Equation (3.16) represents a closed form solution for the particle mass fractions in the frozen Couette flow of a partially ionized symmetric diatomic gas

It is now appropriate to make some comments and comparisons with the special case where ionization is suppressed. Consider first the case of chemical equilibrium flow. At low temperatures where the gas is dissociated only, $c_R = 0$, and equation (3.5) reduces to $c_A = 1/\alpha$, which is exactly that of a dissociated gas. On the other hand, at very high temperatures (e.g., N_2 at $p = 1$ atm., $T \geqslant 9000$°K) $\alpha \cong 1$, and equation (3.5) shows that $c_A \cong 1 - c_R$. Between these temperatures the atom mass fraction must be expressed by equation (3.5). Some authors (see Hansen [20] and Fenster [15] assume that all molecules dissociate into atoms before atoms start to ionize. The above comments show how this assumption is good only at low and high temperatures and that equation (3.5) is more general.

For the frozen flow case equation (3.16a) is exactly that for the atom distribution if ionization is suppressed (see [10]). Hence, εc_{A1} represents the change in atom concentration due to ionization. This result shall be used later when evaluating boundary conditions at the upper wall for the numerical examples presented in this paper.

3.2 *Particle concentrations at the boundaries*

When the flow is in chemical equilibrium particle mass fractions in the gas phase are those corresponding to their equilibrium values as determined by the local temperature, i.e., $c_i = c_{ie}(T)$. At the walls the particle mass fractions depend upon the catalyticity of the wall surface. That is, the particle concentrations at the walls are not necessarily those corresponding to the equilibrium composition. A transition region is, therefore, present. This transition usually takes place, however, in a negligibly thin layer such that the mass fractions at the boundaries can be assumed to correspond to the equilibrium composition at the prescribed wall temperatures (a conventional assumption).

The frozen flow case must, however, be treated differently because species concentration at the walls influences the species distribution throughout the gas. The general case where both walls are treated as "strictly" solid surfaces, and where opposing heterogeneous reactions may occur, shall not be presented here; for treatment of this general case see [21]. We shall here consider the particle concentrations at the upper wall to correspond to the chemical equilibrium composition at the prescribed upper wall temperature. This assumption simulates a boundary layer flow whose free stream is a dissociating and ionizing diatomic gas in chemical equilibrium. Thus, $c_{A\delta} = c_{Ae\delta}$ and $c_{R\delta} = c_{Re\delta}$. It was noted before that c_{A0} corresponds to the case where the gas is dissociated only; hence, the boundary conditions on concentrations at the upper wall can be written as

$$c_{A0\delta} = 1/\alpha_\delta = (1 + 4p/kT_\delta K_D)^{-\frac{1}{2}}, \quad (3.17)$$

$$\varepsilon c_{R1\delta} = (1/\alpha_\delta - 2/\alpha_\delta^2)(1/\alpha_\delta + 1/\alpha_\delta^2)^{\frac{1}{2}} \zeta_\delta^{\frac{1}{2}}, \quad (3.18)$$

$$\varepsilon c_{R1\delta} = (1/\alpha_\delta + 1/\alpha_\delta^2) \zeta_\delta^{\frac{1}{2}}. \quad (3.19)$$

The particle concentrations at the lower wall shall be evaluated by extension of the argument presented by Clarke and McChesney [24] for a dissociated gas. Define Γ_{A_w} as the ratio of the number of atoms converted into molecules to the total number of atoms striking the wall per unit area per unit time; also introduce Γ_{R_w} as the ratio of the number of ion-electron particles converted into neutral particles to the total number of ion-electron particles striking the wall per unit area per unit time. In general, the atom recombination reaction is opposed by the corresponding heterogeneous dissociation reaction, and the neutralization reaction is opposed by the heterogeneous ionization reaction; for this more general treatment see [21]. Here we shall consider the lower wall to be "cool" such that the heterogeneous dissociation and ionization reactions can be neglected. Hence, for the perfect gas mixture considered here ($p = nkT$), the atomic and "ion-electron" mass fluxes into the "cool" lower wall are, respectively (see [21]).

$$-\rho c_A V_A]_w = 2\Gamma_{Aw} c_{Aw} p\,[m_A/2\pi k T_w]^{\frac{1}{2}}/(1 + c_{Aw} + 3c_{Rw}), \quad (3.20)$$

$$-\rho c_R V_R]_w = 2\Gamma_{Rw} c_{Rw} p\,[m_A/2\pi k T_w]^{\frac{1}{2}}/(1 + c_{Aw} + 3c_{Rw}) \quad (3.21)$$

where the proper number density, $n = n_M + n_A + n_I + n_E$, has been used to determine p (which depends upon the random motion of all the species, not their diffusion velocities). Now the constants of integration of equations (3.10) and (3.11) are introduced:

$$-\rho c_A V_A]_w = \tau_w K_1' = 3L\tau_w K_1, \quad (3.22)$$

$$-\rho c_R V_R]_w = \tau_w K_2' = 3L\tau_w K_2. \quad (3.23)$$

Substituting $c_{Aw} = c_{A0w} + \varepsilon c_{A1w}$, $c_{Rw} = \varepsilon c_{R1w}$, $K_1 = K_{10} + \varepsilon K_{11}$ and $K_2 = \varepsilon K_{21}$ into equations (3.22) and (3.23) and equating terms of the same order of magnitude we obtain

$$K_{10} = (4/3)\,\Gamma_{Aw} c_{A0w} p(m_A/2\pi k T_w)^{\frac{1}{2}}/L\rho_\delta u_\delta^2 C_f(1 + c_{A0w}), \quad (3.24)$$

$$\varepsilon K_{21} = (4/3)\Gamma_{Rw}\varepsilon c_{R1w} p(m_A/2\pi k T_w)^{\frac{1}{2}}/L\rho_\delta u_\delta^2 C_f(1 + c_{A0w}), \quad (3.25)$$

and

$$\varepsilon K_{11} = (4/3)\,\Gamma_{Aw}(\varepsilon c_{A1w} - 3\varepsilon c_{R1w} c_{A0w})\, p(m_A/2\pi k T_w)^{\frac{1}{2}}/L\rho_\delta C_f(1 + c_{A0w})^2, \quad (3.26)$$

where we have introduced the coefficient of skin friction, $C_f = \tau_w/(\frac{1}{2})\,\rho_\delta u_\delta^2$. Equation (3.24) and (3.25) and (3.26) together with the expressions for K_{10}, εK_{21} and εK_{11}, noted after equation (3.16) may be solved for c_{A0w}, εc_{A1w} and εc_{R1w} in terms of C_f, T_δ, T_w, p, L and the equilibrium concentrations at the upper wall. Thus, we have obtained all the necessary boundary conditons on concentration. The method of evaluating the concentrations at the boundaries shall be discussed later. We notice, however, that the concentrations at the lower wall depend on the catalyticity of the wall, Γ_{iw}. In general, the value of Γ_{iw} may be anywhere from $\Gamma_{iw} = 0$ to $\Gamma_{iw} = 1$. We shall, however, limit our analysis to the two extreme cases of non-catalytic wall ($\Gamma_{Aw} = \Gamma_{Rw} = 0$) and fully-catalytic wall ($\Gamma_{Aw} = \Gamma_{Rw} = 1$). In the case of a non-catalytic wall it can be shown that all concentrations are constant throughout the layer [21].

4. HEAT TRANSFER AND SKIN FRICTION

The heat transfer rate $(-q_w)$ can be obtained by integrating the energy equation (equation 2.3) and evaluating the constant of integration at the wall;

$$\kappa(\mathrm{d}T/\mathrm{d}y) - \rho\sum_i h_i c_i V_i + \mu\,[\mathrm{d}(u^2/2)/\mathrm{d}y] = -q_w. \quad (4.1)$$

Using $h = \Sigma\, c_i h_i$ with $p =$ constant this becomes

$$\mathrm{d}h/\mathrm{d}y - \sum_i\,[h_i(\mathrm{d}c_i/\mathrm{d}y) + (\rho Pr_f/\mu)\,h_i c_i V_i] + Pr_f[\mathrm{d}(u^2/2)\,\mathrm{d}y] = -\,q_w Pr_f/\mu, \quad (4.2)$$

where $Pr_f = \mu c_{pf}/\kappa$ is the frozen Prandtl number, considered constant†, in this analysis; $c_{pf} = \sum_i c_i c_{pi}$ is the "frozen" specific heat of the gas mixture. Integrating from $y = 0$, using $\int_0^y \mathrm{d}y'/\mu = u/\tau_w$, replacing $\rho_i V_i$ by equations (2.11a) and (2.11b) and noting that $h_0^D \cong h_A$ and $h_R - h_M = h_0^D + h_0^I + c_{pA}T$ (see Appendix), we obtain

$$h - h_w + \int_{c_{Aw}}^{c_A} (R_1 - TR_2)\,\mathrm{d}c_A' + \int_{c_{Rw}}^{c_R} (R_3 + TR_4)\,\mathrm{d}c_R' + Pr_f(u^2/2) = -\,q_w Pr_f u/\tau_w, \quad (4.3)$$

where R_1, R_2, R_3, and R_4 are known functions of c_A and c_R, and are proportional to $\rho\mathscr{D}_{AM}Pr_f/\mu$. Carrying the integration to $y = \delta$, equation (4.3) yields

$$-\,q_w = (1/2)\,(\rho_\delta u_\delta C_f/Pr_f)\,[h_\delta - h_w + \int_{c_{Aw}}^{c_{A\delta}} (R_1 - TR_2)]_d c_A + \int_{c_{Rw}}^{c_{R\delta}} (R_3 + TR_4)\,\mathrm{d}c_R + (1/2)\,Pr_f u_\delta^2]. \quad (4.4)$$

The effect of ionization on the heat transfer rate $(-q_w)$ is most apparent in the second integral of equation (4.4); when

$$c_R = 0, \int_{c_{Aw}}^{c_{A\delta}} (R_1 - TR_2)\,\mathrm{d}c_A$$

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† The frozen Prandtl number has been evaluated in [21] for 500°K < T < 12 000°K where it is shown to vary between $0{\cdot}4 < Pr_f < 0{\cdot}7$; it is taken to be 0·6 in this study.

reduces to

$$h_0^D \int_{c_{Aw}}^{c_{A\delta}} (\rho \mathscr{D}_{AM} Pr_f/\mu - 1)\, \mathrm{d}c_A;$$

then equation (4.4) becomes exactly that obtained by Enkenhus [3] for a dissociated diatomic gas. One also notes that the ion-electron particles transfer their dissociation and ionization energy as well as the translational energy ($c_{p_A}T$) which is due to electrons (see Appendix). The last term in equation (4.4) is, of course, the viscous heating term.

When equations (4.3) and (4.4) are combined a useful relation between temperature, mass fractions and velocity results;

$$\begin{aligned} h - h_w &+ \int_{c_{Aw}}^{c_A} (R_1 - TR_2)\, \mathrm{d}c_A + \int_{c_{Rw}}^{c_R} (R_3 \\ &+ TR_4)\, \mathrm{d}c_R + (1/2)\, Pr_f u_\delta^2 (u/u_\delta)^2 \\ &= (u/u_\delta)\, [h_\delta - h_w + \int_{c_{Aw}}^{c_{A\delta}} (R_1 - TR_2)\, \mathrm{d}c_A \\ &+ \int_{c_{Rw}}^{c_{R\delta}} (R_3 + TR_4)\, \mathrm{d}c_R + (1/2)\, Pr_f u_\delta^2]. \qquad (4.5) \end{aligned}$$

This expression, when using the mass fraction-temperature relation (equations 3.7 and 3.8) for chemical equilibrium flow or the mass fraction-velocity relation, equation (3.16) for the frozen flow, yields a useful temperature-velocity relation. The use of equation (4.5) in solving the Couette flow problem shall be discussed later.

The skin friction coefficient can be obtained by integrating equation (2.2) using $C_f = \tau_w/(1/2)\,\rho_\delta u_\delta^2$ and $Re_\delta = \rho_\delta u_\delta \delta/\mu_\delta$;

$$C_f = (2/Re_\delta) \int_0^1 (\mu/\mu_\delta)\, \mathrm{d}(u/u_\delta). \qquad (4.6)$$

The recovery enthalpy is defined as the value of h_w when $q_w = 0$ (adiabatic wall); from equation (4.4) we obtain

$$\begin{aligned} h_r = h_\delta &+ \int_{c_{Ar}}^{c_{A\delta}} (R_1 - TR_2)\, \mathrm{d}c_A \\ &+ \int_{c_{Rr}}^{c_{R\delta}} (R_3 + TR_4)\, \mathrm{d}c_R + (1/2)\, Pr_f u_\delta^2, \qquad (4.7) \end{aligned}$$

where c_{Ar}, c_{Rr} are respectively the atom and the ion-electron mass fractions at the adiabatic wall.

Now define the enthalpy recovery factor as $\theta = (h_r - h_\delta)/(1/2)\, u_\delta^2$; from equation (4.7) we find

$$\begin{aligned} \theta = Pr_f + (2/u_\delta^2)\, [&\int_{c_{Ar}}^{c_{A\delta}} (R_1 - TR_2)\, \mathrm{d}c_A \\ &+ \int_{c_{Rr}}^{c_{R\delta}} (R_3 + TR_4)\, \mathrm{d}c_R]. \qquad (4.8) \end{aligned}$$

In additon to its dependence on atom mass fractions at the walls, we see that the recovery enthalpy depends also on the ion-electron mass fraction at the wall. When both $c_A = 0$ and $c_R = 0$, equation (4.8) yields the well known result $\theta = Pr_f$.

A relationship between skin friction and the heat transfer coefficient, defined by $St = -q_w/\rho_\delta u_\delta (h_r - h_w)$, may be obtained from equation (4.4) by using equation (4.7); it is

$$\begin{aligned} St/C_f = (1/2\, Pr_f)\, \{1 + (h_r - h_w)^{-1}\, [&\int_{c_{Aw}}^{c_{Ar}} (R_1 \\ - TR_2)\, \mathrm{d}c_A + &\int_{c_{Rw}}^{c_{Rr}} (R_3 + TR_4)\, \mathrm{d}c_R]\}. \qquad (4.9) \end{aligned}$$

As was found before for the dissociated gas [3], the simple relation $St/C_f = 1/2\, Pr_f$ does not hold any longer when the gas is ionized.

A parameter that is useful for expressing the heat transfer rate is the Nusselt number defined by $Nu = St Pr_f Re_\delta$; it is obtained from equation (4.9) as

$$\begin{aligned} Nu = (1/2) C_f Re_\delta \{1 + (h_r - h_w)^{-1}\, [&\int_{c_{Aw}}^{c_{Ar}} (R_1 \\ - TR_2)\, \mathrm{d}c_A + &\int_{c_{Rw}}^{c_{Rr}} (R_3 + TR_4)\, \mathrm{d}c_R]\}. \qquad (4.10) \end{aligned}$$

5. COMPLETE FLOW SOLUTIONS, NUMERICAL CALCULATIONS AND DISCUSSIONS OF THE RESULTS

5.1 *Chemical equilibrium flow*

When the gas is in chemical equilibrium the particles mass fractions are functions of temperature only and equation (4.5) can be written as

$$h - h_w + \int_{T_w}^{T} [(R_1 - T'R_2)(\mathrm{d}c_A/\mathrm{d}T') + (R_3 + T'R_4)(\mathrm{d}c_R/\mathrm{d}T')]\,\mathrm{d}T' + (1/2)\,Pr_f u_\delta^2 (u/u_\delta)^2$$
$$= (u/u_\delta)\,\{h_\delta - h_w + \int_{T_w}^{T_\delta} [(R_1 - TR_2)(\mathrm{d}c_A/\mathrm{d}T) + (R_3 + TR_4)(\mathrm{d}c_R/\mathrm{d}T)]\,\mathrm{d}T + (1/2)\,Pr_f u_\delta^2\}. \tag{5.1}$$

Using the expression for h, obtained in the appendix in terms of c_A, c_R and T, and using equations (3.7) and (3.8), a relation between T and u/u_δ is obtained:

$$h - h_w + \int_{c_{Aw}}^{c_A} (R_1 - TR_2)\,\mathrm{d}c_A + \int_{c_{Rw}}^{c_R} (R_3 + TR_4)\,\mathrm{d}c_R + (\tfrac{1}{2})\,Pr_f u_\delta^2 (u/u_\delta)^2$$
$$= (u/u_\delta)\,[h_\delta - h_w + \int_{c_{Aw}}^{c_{A\delta}} (R_1 - TR_2)\,\mathrm{d}c_A + \int_{c_{Rw}}^{c_{R\delta}} (R_3 + TR_4)\,\mathrm{d}c_R + (\tfrac{1}{2})\,Pr_f u_\delta^2],$$

where

$$R_1 = (\rho\mathscr{D}_{AM}Pr_f/\mu)\,\{h_0^D[1 + 2c_R(1 - c_A + c_A^2 + c_A^3)/(1 - c_A + c_A^2 + 3c_A^3)]$$
$$- (h_0^D + h_0^I)\,c_R(1 - c_R^2)/(1 - c_A + c_A^2 + 3c_A^3)\} - h_0^D,$$

$$R_2 = c_{pA}(\rho\mathscr{D}_{AM}Pr_f/\mu)\,c_R(1 - c_A^2)/(1 - c_A + c_A^2 + 3c_A^3),$$

$$R_3 = (\rho\mathscr{D}_{AM}Pr_f/\mu)\,[(h_0^D + h_0^I)(1 + c_A)^3/(1 - c_A + c_A^2 + 3c_A^3)$$
$$- 2h_0^D(c_A + c_A^4)/(1 - c_A + c_A^2 + 3c_A^3)] - (h_0^D + h_0^I),$$

$$R_4 = c_{pA}(\rho\mathscr{D}_{AM}Pr_f/\mu)\,(1 + c_A)^3/(1 - c_A + c_A^2 + 3c_A^3).$$

Having obtained the T vs. u/u_δ relationship, the skin friction is obtained by integrating equation (4.6). The velocity distribution, and consequently the temperature distribution, is obtained from

$$y/\delta = (2/Re_\delta C_f) \int_0^{u/u_\delta} (\mu/\mu_\delta)\,\mathrm{d}(u/u_\delta)', \tag{5.2}$$

which results from integration of equation (2) from $y = 0$.

The recovery temperature is obtained by a trial and error procedure based on equation (A.1) of the Appendix. The recovery temperature is then used in equations (4.8) and (4.10) to obtain the enthalpy recovery factor and the Nusselt number respectively.

5.2 *Frozen flow*

Particle concentrations at the upper wall are obtained from equations (3.17), (3.18) and (3.19).

A value for C_f is assumed and c_{A_w} is calculated by iteration from an equation resulting from equating equation (3.24) with K_{10} following equation (3.16c). Then εc_{A1w} and c_{Rw} are calculated from their respective equations mentioned earlier. A T vs. u/u_δ relationship is then obtained by assuming an initial relation and iterating by using equation (4.5). This relationship is then used to integrate the equation

$$C_f = (2/Re_\delta) \int_0^1 (u/u_\delta)\,\mathrm{d}(u/u_\delta)$$

which results from equation (5.2). The new value of C_f is then compared with the assumed C_f and the whole process is repeated until convergence is achieved. For further details see [21].

The recovery temperature is computed by assuming a wall temperature T_r; boundary concentrations and a temperature vs. velocity relation are then obtained. This T vs. u/u_δ relation is then used in equation (4.7) written as

$$\begin{aligned} c_{pA}T_r = \{ & h_\delta - (1/2)\,Pr_f u_\delta^2 - h_0^D(c_{Ar} + c_{Rr}) \\ & - (h_0^D + h_0^I)\,c_{Rr} + \int_0^1 [(R_1 - TR_2)\,\mathrm{d}c_A/\mathrm{d}(u/u_\delta) \\ & + (R_3 + TR_4)\,\mathrm{d}c_R/\mathrm{d}(u/u_\delta)]\,\mathrm{d}(u/u_\delta)\} \\ & \times [(0{\cdot}7 + 0{\cdot}2\,\psi_r)(1 - c_{Ar} - c_{Rr}) + c_{Ar} \\ & \qquad + 2c_{Rr}]^{-1} \end{aligned}$$

The resulting recovery temperature is used again and the whole process is repeated until T_r converges. Nusselt number and the recovery enthalpy factor are obtained from equations (4.10) and (4.8).

5.3 *Numerical calculations and discussion of the results*

Calculations performed here are for nitrogen, which has a dissociation energy of 9·756 eV/atom. The vibrational temperature for nitrogen is T_v = 3400°K and the value for L was found by Fay and Kemp to be approximately 0·6. The upper wall temperature is taken to be 12000°K and the lower wall temperature is 500°K. The results presented here in graphical form are compared on the same graphs with the results obtained for the flow, if ionization were not to occur; that is, when ionization is artifically suppressed. The reason for doing so is that we wish to know explicitly what are the effects of ionization on the flow. The flow with ionization suppressed was thus treated under the same boundary conditions, using the same thermodynamic and transport properties, except for c_R which was set equal to zero.

All the numerical calculations were carried out on a CDC–3400 digital computer.†

The velocity distribution is shown in Fig. 1; the steeper velocity gradient of the ionized gas flow is due to the reduction in the viscosity of the diatomic gas when it becomes ionized.

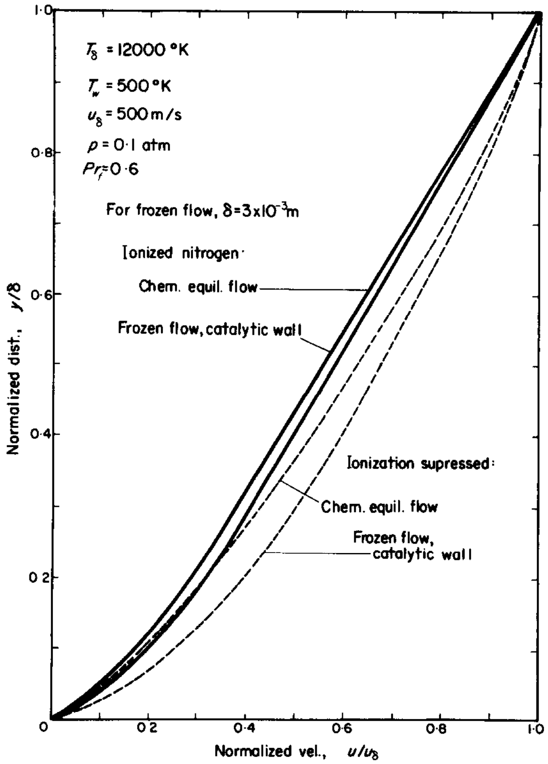

FIG. 1. Velocity distribution for the couette flow of nitrogen; fully-catalytic wall for the frozen flow cases.

The temperature distribution is shown in Fig. 2. When the gas phase is in chemical equilibrium the temperature gradient near the "hot" upper wall is small. This indicates that heat transfer in this region is mainly due to diffusion, for as we move towards the lower wall ionized particles are neutralized, releasing their ionization energy. Part of this energy is carried by thermal conduction and we note that the temperature gradient becomes steeper as the number of ionized particles decreases. When most of the ionized particles have been neutralized to atoms and these have now reached a maximum concentration, the energy released by neutralization is carried primarily by thermal conduction, and results in the steep temperature gradient seen at $T/T_w \cong 14$. This is followed, moving towards the lower wall, by a region where atoms are now diffusing towards the lower wall, as explained earlier. The energy transfer is now by thermal conduction and atom diffusion, hence, the reduction in the temperature gradient. Close to the lower wall most

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† The authors gratefully acknowledge the machine time granted by the Vogelback Computing Center of North-Western University.

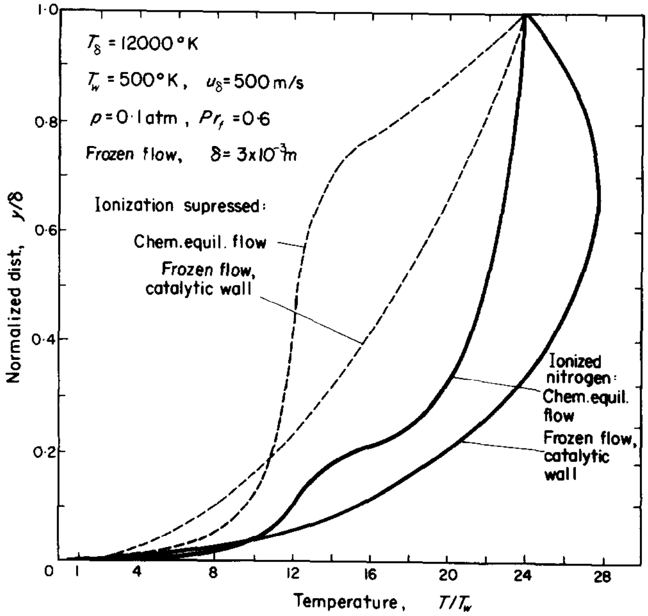

FIG. 2. Temperature distribution for the couette flow of nitrogen; fully-catalytic wall for the frozen flow cases.

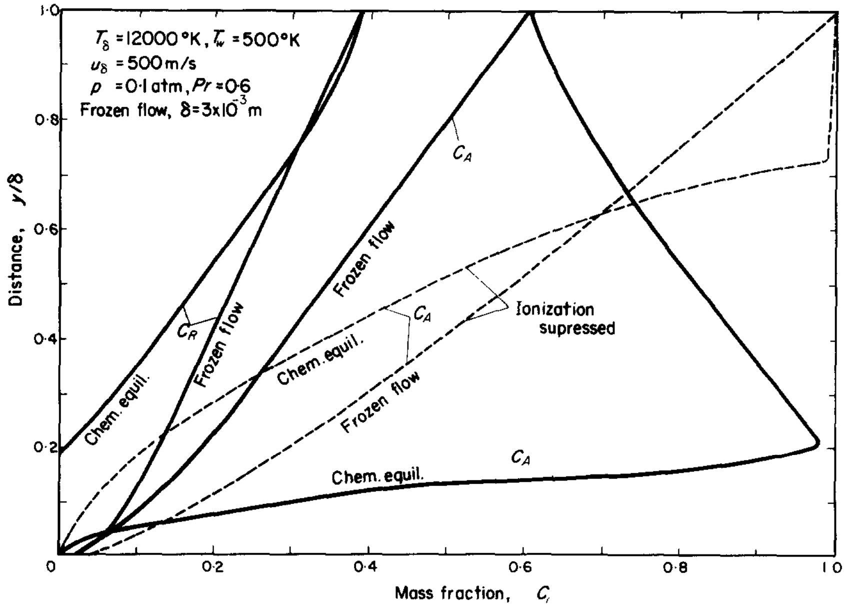

FIG. 3. Specie mass fraction distributions for the couette flow of nitrogen; fully-catalytic wall for the frozen flow cases.

atoms have recombined into molecules and released their dissociation energy. Energy transfer in this region is by thermal conduction alone; this explains the steep temperature gradient near the lower wall. In comparison with the case where ionization is suppressed one can then expect ionization to increase the rate of heat transfer.

One may also note the linear temperature gradient near the upper wall when ionization is suppressed. This is because the gas in this region has reached full dissociation, and energy is transferred by thermal conduction alone.

When the gas phase is chemically frozen, and the lower wall is fully catalytic, the temperature distribution exhibits a maximum at a short distance from the "hot" wall. This is because the total heat flux is constant with y, and near the upper wall the heat flux is carried mainly by diffusion such that excess heat generated by viscous dissipation must be carried back to the upper wall by thermal conduction. This may be amplified as follows: by using equations (A.3) and (A.4), equation (4.1) can be written as

$$\kappa \, \mathrm{d}T/\mathrm{d}y + C_1 T + u\tau_w = C_2,$$

where the constants, though different in magnetude, are positive since $q_w < 0$. At the lower wall the sign of $(\mathrm{d}T/\mathrm{d}y)_w$ depends on T_w and the constants. In the computations carried out here $(\mathrm{d}T/\mathrm{d}y)_w$ is found to be positive. As y increases (and u increases) T and κ increase and $\mathrm{d}T/\mathrm{d}y$ must decrease. This continues until $\mathrm{d}T/\mathrm{d}y = 0$. A further increase in u causes $\mathrm{d}T/\mathrm{d}y$ to become negative. Further details may be found in [21].

The mass fraction distributions are shown in Fig. 3. As explained previously, the atom mass fraction, when the gas phase is in chemical equilibrium, reached a maximum somewhere in the flow layer. This position marks the change in the direction of atomic mass flux. When ionization is suppressed the atom mass fraction reaches unity at some distance away from the "hot" wall (there is no local maximum; see Fig. 3). Due to the low pressure ($p = 0{\cdot}1$ atm) and the requirement of chemical equilibrium at the upper wall, c_R is not much smaller than c_A for this particular numerical example. Thus, the linearized solution is a poorer approximation near the upper wall.

Ionization increases the value of Nusselt number, for chemical equilibrium flow, by 2·8 times as much as its value when ionization is suppressed. This is shown in Fig. 4. However, one must note that the increase in the Nusselt number is partly due to the increase in the Reynolds number which, in turn, is due to the decrease in the viscosity. Calculations were carried out for the same Re_δ (for ionized and suppressed ionization flows); we found ionization to increase Nu at $T_\delta = 12\,000$°K by 100 per cent at $T_\delta = 11\,000$°K by 49 per cent and at $T_\delta = 10\,000$°K by 20 per cent; that is, the heat transfer rate increases with the increase in the degree of ionization. When the gas phase is chemically frozen ionization also increases the

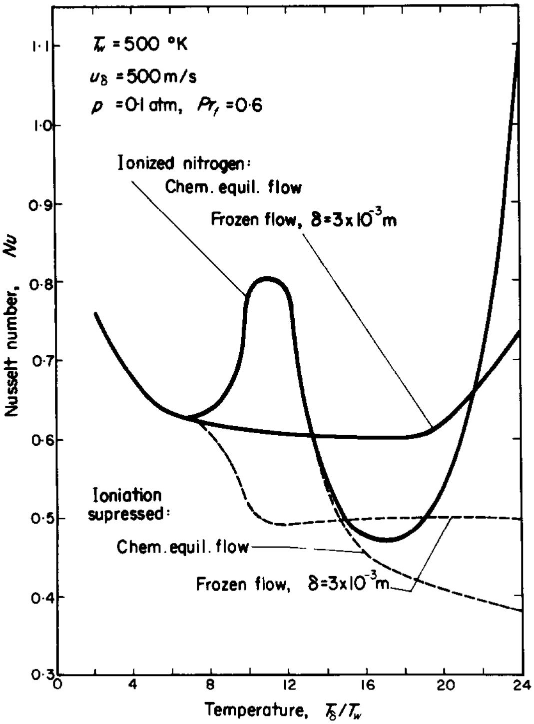

FIG. 4. Effect of ionization on heat transfer for the couette flow of nitrogen; fully-catalytic wall for the frozen flow cases.

heat transfer rate, but not as much as in chemical equilibrium flow. This is because near the lower wall the ion-electron mass fraction is significant (see discussion concerning Fig. 2), and part of the ionization energy is retained; hence, energy is not completely recovered.

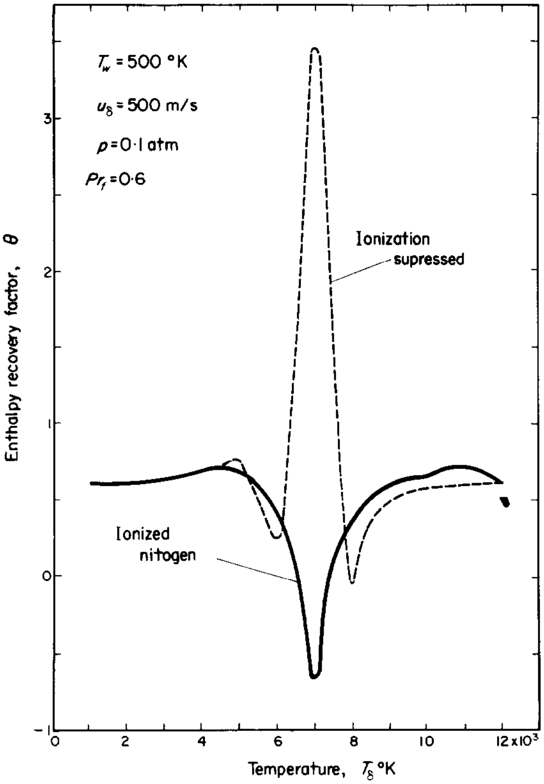

FIG. 5. Effect of ionization on enthalpy recovery factor for the couette flow of nitrogen; chemical-equilibrium flow.

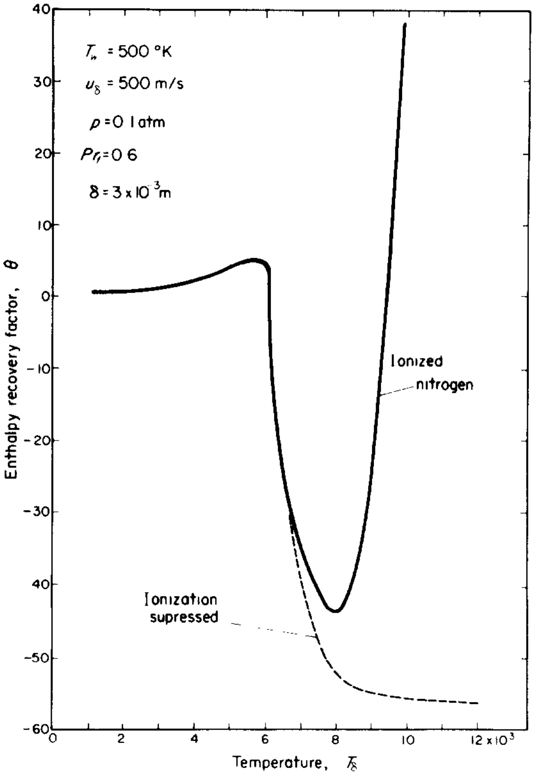

FIG. 6. Effect of ionization on enthalpy recovery factor for the couette flow of nitrogen; frozen flow with fully-catalytic wall.

The effect of ionization on the enthalpy recovery factor is shown in Figs. 5 and 6 respectively, for chemical equilibrium and frozen flow. The behaviour of this parameter can be best explained with the help of equation (4.8). When the flow is in chemical equilibrium and at low values of the upper wall temperature, $c_{A\delta} = c_{Ar} = 0$ and $c_{R\delta} = c_R = 0$; then $\theta = Pr_f$. With increase in T_δ dissociation occurs, but $c_{Ar} > c_{A\delta}$ due to viscous heating (c_R is still zero), and θ starts to decrease. At higher temperatures where ionization occurs at the upper wall, viscous dissipation may increase the ion-electron concentration at the adiabatic wall, i.e., $c_{Rr} > c_{R\delta}$ but $c_{Ar} < c_{A\delta}$; however, since atoms are dominant in the flow, θ starts to increase. At very high upper wall temperatures the energy contribution due to viscous dissipation is relatively small so that $c_{A\delta} \cong c_{Ar}$ and $c_{R\delta} = c_{Rr}$; thus θ approaches Pr_f again. When the flow is chemically frozen and the upper wall is at low temperatures, θ is also equal to Pr_f. At temperatures where ionization occurs at the upper wall, it was found that $T_r > T_\delta$, yet the catalytic wall reduces the particle mass fractions such that $h_r < h_\delta$; this causes θ to decrease.

The effect of ionization on the parameter $(1/2)Re_\delta C_f$ is shown in Fig. 7. The increase in this parameter is mainly due to the increase in Re_δ; that is, ionization produces a small effect on the skin friction coefficient.

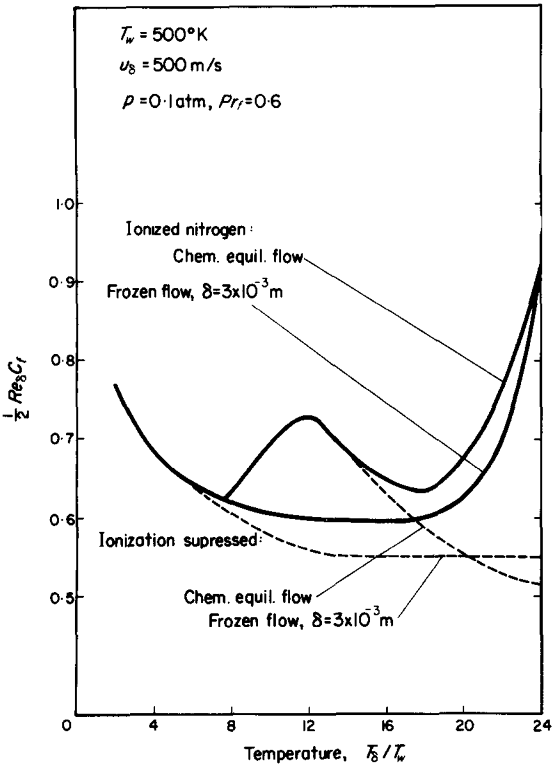

FIG. 7. Effect of ionization on skin friction for the couette flow of nitrogen; fully-catalytic wall for the frozen flow case.

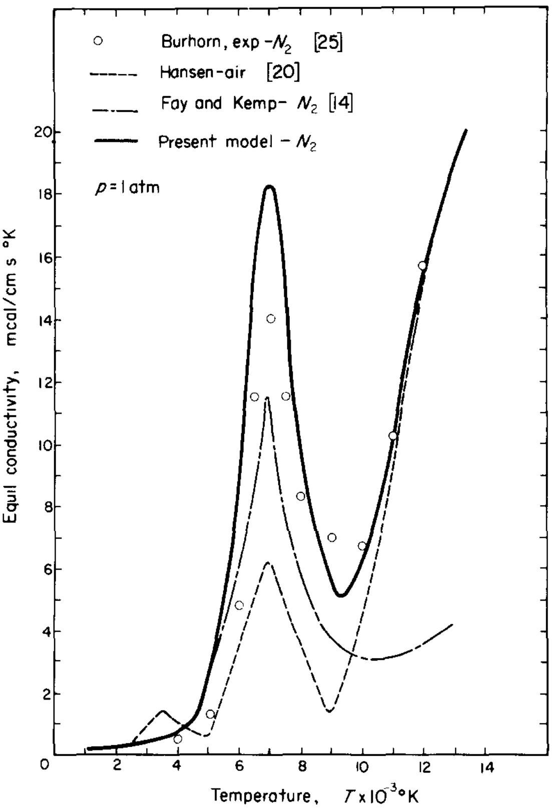

FIG. 8. Equilibrium conductivity of nitrogen.

Figure 8 shows the "equilibrium" thermal conductivity calculated by the model developed in this paper. In the same figure Burhorn's [25] experimental points, Fay and Kemp's [14] calculations and Hansen's [20] calculations are presented. The present model appears to give better agreement with the experimental results except for $5000°K \leqslant T \leqslant 7000°K$. This is because we have used an approximate formula for the ionization equilibrium constant given by Duclos *et al.* [26] (see Appendix). This formula over-estimates the number of ionized particles in the flow for $T \leqslant 7000°K$, by a maximum of 15 per cent. Our reasonable agreement with Burhorn's results lends support to the model and the theory developed in this paper.

Fay and Kemp's "equilibrium" thermal conductivity calculated by use of their theory that *neglected* radiation, was compared by them, and found in good agreement, with Yos "equilibrium" thermal conductivity calculated by a theory that included radiation. When radiation is neglected the value of "equilibrium" thermal conductivity is large and Yos' calculations with radiation terms removed are then in good agreement with Maecker's [27] measurements (note that Maecker's measurements agree with Burhorn's measurements). Thus, one can conclude that use of Fay and Kemp's binary model would result in an underestimate in the calculation of the "equilibrium" thermal conductivity as well as the heat transfer rate.

As the pressure increases ionization is reduced and the ion-electron mass fractions become much smaller than the atom mass fraction in which case the theory of this paper gives a much better approximation. Also we may note that the theory of this paper gives better approximations for smaller upper wall temperatures. For the case shown here ($T_\delta = 12\,000°K$, $p = 0{\cdot}1$ atm), the approximations become less valid near the upper wall.

Hansen's calculations, one may note, are for air which at high temperature may form NO; this was neglected by Hansen and this theory was developed under the assumption that oxygen and nitrogen molecules are completely dissociated before ionization commences. Also, when the gas is ionized his mixture consisted of N, N^+, 0, 0^+ and E. The assumption of independent dissociation and ionization reactions may be the cause of the low "equilibrium" thermal conductivity for $T \leqslant 9000°K$ as shown in Fig. 8.

6. CONCLUSIONS

(1) The effects of diffusion in partially ionized symmetric diatomic gases may be accounted for by treating the fluid as a ternary gas mixture containing molecules, atoms, and ion-electron species.

(2) In addition to the atomic mass flux, ionization produces an ion-electron mass flux flowing towards the "cool' wall; when the gas phase is in chemical equilibrium ionization causes the atomic mass flux to split into two streams flowing towards the "cool" wall and the "hot" wall.

(3) When the gas is thermally ionized higher temperatures are found in most of the flow layer as compared with the results when ionization is suppressed.

(4) The rate of heat transfer increases with the degree of ionization and is larger when the gas phase is in chemical equilibrium than when it is chemically frozen.

(5) The effects of ionization cannot be reasonably estimated by extrapolation of results obtained by suppressing ionization.

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APPENDIX

Thermodynamic and Transport Properties

The specific heats of the different species of the ionized diatomic gas mixture were given by Fay and Kemp [14]; from these we calculated the enthalpy of the gas mixture as [21]

$$h = \sum_{i=1}^{i=3} c_{pi}c_i = [(0{\cdot}7 + 0{\cdot}2\,\psi)(1 - c_A - c_R) + c_A + 2c_R]\, c_{pA}T + H_0^D(c_A + c_R) + h_0^I c_R. \qquad (A.1)$$

and

$$h_A - h_M = h_0^D + c_{pA}T(0{\cdot}3 - 0{\cdot}2\,\psi); \qquad (A.2)$$

where

$$\psi = (T_v/T)/[\exp.(T_r/T) - 1].$$

The quantity $c_{pA}T(0{\cdot}3 - 0{\cdot}2\,\psi)$ was calculated for nitrogen and oxygen up to $T = 12\,000$°K and found to have a maximum of $0{\cdot}06\,h_0^D$; hence, we considered

$$h_A - h_M \simeq h_0^D. \qquad (A.3)$$

By requiring the mixture of molecules, atoms, and ion-electron particles to have the same enthalpy as the mixture of molecules, atoms, ions and electrons, and by using equation (A.3) we obtain

$$h_R - h_M \cong h_0^D + h_0^I + c_{pA}T, \qquad (A.4)$$

where $c_{pA}T$ appears due to the presence of electrons.

The equation of state for the mixture is

$$p = kT\sum_i n_i = (\rho kT/2m_A)(1 + c_A + 3c_R). \qquad (A.5)$$

The viscosity of the partially ionized diatomic gas was obtained by Fay and Kemp under the assumptions that ions and electrons contribute nothing to the viscosity; when their formula is written in terms of mass fraction it is

$$\mu = \mu_A(1 + c_A + c_R)^{-1}[0{\cdot}82 + 1{\cdot}18(c_A + c_R)]\,[(1 + c_A - c_R)/(1 + c_A + 3c_R)]. \qquad (A.6)$$

The ratio in the last set of square brackets represents a correction factor accounting for neglecting the contributions of ions and electrons; it is the mole fraction of atoms and molecules in the ionized gas mixture. Although it is true that electrons would contribute negligibly to the viscosity, due to their small mass, ions cannot be neglected because their mass is essentially equal to that of atoms; hence, we shall not neglect the contributions of ions; the correction factor in equation (A.6) then becomes the molecule, atom and ion mole fraction, and equation (A.6) becomes

$$\mu = \mu_A[0{\cdot}82 + 1{\cdot}18(c_A + c_R)]/(1 + c_A + 3c_R). \qquad (A.7)$$

It may be noted that the viscosities obtained by equations (A.6) and (A.7) were calculated [21] and compared with Hansen's viscosity; it was found that equation (A.7) is to be preferred over equation (A.6).

The approximate functional forms for the dissociation and ionization equilibrium constants used in the numerical calculations are

$$K_D = A\exp.(-\theta_D/T), \qquad (A.8)$$

$$K_I = DT^m\exp.(-\theta_I/T), \qquad (A.9)$$

where A, D and m are constants. Equation (A.9) was suggested by Duclos *et al.* [25] with $m = 5/4$ for nitrogen. Both equations (A.8) and (A.9) were used to calculate the equilibrium composition for oxygen and nitrogen. In comparison with Drellishak's [22] data equation

(A.8) gives excellent agreement; equation (A.9) is in good agreement for $T > 5000°\text{K}$. For nitrogen gas we found $A = 1{\cdot}14 \times 10^{25}\ \text{cm}^3$, $D = 0{\cdot}97 \times 10^{17}\ \text{cm}^3/°\text{K}$. θ_D and θ_I are the characteristic temperatures for dissociation and ionization for nitrogen gas; they are respectively 113 000°K and 168 000°K.

Résumé—L'écoulement de Couette d'un gaz diatomique symétrique partiellement ionisé est étudié à la fois pour l'écoulement en équilibre chimique et pour l'écoulement chimiquement figé; les résultats représentent une généralisation directe de l'écoulement de Couette avec seulement de la dissociation. On insiste sur la mise en lumière du rôle de l'ionisation au moyen de cette géométrie relativement simple, et de l'introduction d'un nouveau modèle ternaire pour la diffusion. On considère que le mélange gazeux consiste en molécules et en éledtrons, sauf pour les effets de la diffusion où l'on suppose que les ions et les électrons diffusent ensemble comme une seule espèce (diffusion ambipolaire). La théorie est limitée aux écoulements dans lesquels les densités numériques d'ions et d'électrons, bien qu'importantes, sont faibles par rapport à celle des particules neutres et la théorie est conduite en linéarisant par rapport aux fractions massiques ion-électron. Les équations linéarisées sont résolues analytiquement et les résultats numériques sont présentés pour l'azote. On trouve que l'ionisation produit un flux massique ion-électron allant de la paroi supérieure "chaude" à la paroi inférieure "froide"; lorsque la phase gazeuse est en équilibre chimique, l'ionisation conduit le flux massique atomique à se scinder en deux écoulements, l'un diffusant vers la paroi supérieure "chaude" et l'autre vers la paroi inférieure "froide". L'ionisation a un effet important sur la distribution de température et sur l'enthalpie de frottement. Le flux de chaleur transmis croît avec le degré d'ionisation.

Zusammenfassung—Die Couette-Strömung eines teilweise ionisierten symmetrisch-zweiatomigen Gases wird untersucht, sowohl für das chemische Gleichgewicht, als auch für chemisch eingefrorene Strömung; die Ergebnisse liefern eine direkte Verallgemeinerung der Couette-Strömung nur mit Ionisation. Die Überlegungen konzentrieren sich auf die Ionisation anhand der vorliegenden ziemlich einfachen Geometrie, und auf die Einführung eines neuen Modells für die Diffusion von drei Komponenten. Das Gasgemisch ist aus folgenden Komponenten bestehend gedacht: symmetrische zweiatomige Moleküle, (atomare) Ionen und Elektronen. Für die Diffusionseffekte sind allerdings die Ionen und Elektronen als eine Einheit diffundierend gedacht (ambipolare Diffusion). Die Theorie ist begrenzt auf Strömungen, mit kleiner Dichte der Ionen oder Elektronen im Vergleich zur Dichte der neutralen Teilchen. Die analytische Behandlung ist linearisiert in Bezug auf den Massenanteil der Ionen und Elektronen. Die linearisierten Gleichungen sind analytisch gelöst, und für Stickstoff sind numerische Ergebnisse wiedergegeben. Es zeigt sich, dass die Ionisation einen Ionen-Elektronen-Massenstrom von der oberen "heissen" Wand zur unteren "kalten" Wand hervorruft. Wenn das Gas in chemischem Gleichgewicht ist, ruft die Ionisation eine Aufspaltung des atomaren Massenstromes in zwei Ströme hervor, wovon einer zur oberen "heissen" Wand und der andere zur unteren "kalten" Wand diffundiert. Die Ionisation hat eine deutliche Auswirkung auf die Temperaturverteilung und die Eigentemperatur. Der Wärmeübergang steigt mit dem Grade der Ionisation.

Аннотация—Исследуется течение Куэтта частично ионизированного симметричного двухатомного газа для случаев химически равновесного и химически замороженного течений; результаты представлены в виде прямого обобщения течения Куэтта при наличии только диссоциации. Основное внимание обращается на освещение роли ионизации путем использования этой относительно простой геометрии и введения новой тройной модели диффузии. Предполагается, что газовая смесь состоит из симметричных двухатомных молекул, атомов и ионов (атомных) и электронов, за исключением явлений диффузии, где предполагается, что ионы и электроны диффундируют как одно целое (амбиполярная диффузия). Теория ограничена течением, где плотность числа ионов и электронов, хотя и довольно значительная, все же мала по сравнению с нейтральными частицами; анализ имеет линейный характер относительно массовых концентраций ионов-электронов. Линеаризованные уравнения решаются аналитически. Представлены численные результаты для азота. Установлено, что ионизация вызывает массовый ионно-электронный поток по направлению от верхней «горячей» стенки к нижней «холодной». При химическом равновесии газовой фазы ионизация вызывает расслоений массового потока атомов на два, один из которых диффундирует к верхней «горячей» стенке, а другой к нижней — «холодной». Ионизация оказывает значительное влияние на распределение температуры и энтальпию восстановления. Скорость теплообмена возрастает со степенью ионизации.